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NAME ______________ DATE ________ PERIOD ____

8 Anticipation Guide

Rational Expressions and Equations

Step 1 ***Before you begin Chapter 8***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. Since a denominator cannot equal 0, the expression $\frac{3x^2(x-1)}{x+5}$ is undefined for $x = -5$.	A
	2. To divide two rational expressions, multiply by the reciprocal of the divisor.	A
	3. The least common multiple of three monomials is found by multiplying the monomials together.	D
	4. Before adding two rational expressions, a common denominator must be found.	A
	5. The graph of a rational function containing an asymptote will be symmetric over the asymptote.	D
	6. Since $f(x) = \frac{(m+4)(m-2)}{m+4}$ can be simplified to $f(x) = m - 2$, the graph of $f(x)$ will be the straight line defined by $y = m - 2$.	D
	7. $y = kxyz$ is an example of a joint variation if k, x, y, and z are all not equal to 0.	A
	8. The shape of the graph of $y = -3x^2 - 2x + 4$ can only be determined by graphing the function.	D
	9. Because the graph of an absolute value function is in the shape of a V, the graph of $y = \|x\| + 4$ will also be in the shape of a V.	A
	10. When solving rational equations, solutions that result in a zero in the denominator must be excluded.	A

Step 2 ***After you complete Chapter 8***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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NAME ______________ DATE ________ PERIOD ____

8-1 Lesson Reading Guide

Multiplying and Dividing Rational Expressions

Get Ready for the Lesson

Read the introduction to Lesson 8-1 in your textbook.

- Suppose that the Goodie Shoppe also sells a candy mixture made with 4 pounds of chocolate mints and 3 pounds of caramels, then $\frac{4}{7}$ of the mixture is mints and $\frac{3}{7}$ of the mixture is caramels.
- If the store manager adds another y pounds of mints to the mixture, what fraction of the mixture will be mints? $\frac{4+y}{7+y}$

Read the Lesson

1. a. In order to simplify a rational number or rational expression, **factor** the numerator and **denominator** and divide both of them by their **greatest common factor**.

b. A rational expression is undefined when its **denominator** is equal to **0**. To find the values that make the expression undefined, completely **factor** the original **denominator** and set each factor equal to **0**.

2. a. To multiply two rational expressions, **multiply** the **numerators** and multiply the denominators.

b. To divide two rational expressions, **multiply** by the **reciprocal** of the **divisor**.

3. a. Which of the following expressions are complex fractions? **ii, iv, v**

i. $\frac{7}{12}$ **ii.** $\frac{\frac{3}{8}}{\frac{5}{16}}$ **iii.** $\frac{r+5}{r-5}$ **iv.** $\frac{\frac{z+1}{z}}{z}$ **v.** $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

b. Does a complex fraction express a multiplication or division problem? **division** How is multiplication used in simplifying a complex fraction? **Sample answer: To divide the numerator of the complex fraction by the denominator, multiply the numerator by the reciprocal of the denominator.**

Remember What You Learned

4. One way to remember something new is to see how it is similar to something you already know. How can your knowledge of division of fractions in arithmetic help you to understand how to divide rational expressions? **Sample answer: To divide rational expressions, multiply the first expression by the reciprocal of the second. This is the same "invert and multiply" process that is used when dividing arithmetic fractions.**

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NAME ______________________ DATE __________ PERIOD _____

8-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Example **Simplify each expression.**

a. $\frac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{b}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s}$

$$\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s} = \frac{\cancel{3} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{s} \cdot s \cdot s \cdot 2 \cdot 2 \cdot \cancel{5} \cdot \cancel{t} \cdot \cancel{t}}{\cancel{5} \cdot \cancel{t} \cdot \cancel{t} \cdot t \cdot t \cdot \cancel{3} \cdot 3 \cdot \cancel{r} \cdot \cancel{r} \cdot r \cdot \cancel{s}} = \frac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \frac{4s^2}{3rt^2}$$

c. $\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{\cancel{(x + 4)}(x + 4)\cancel{(x - 1)}}{2\cancel{(x - 1)}(x - 2)\cancel{(x + 4)}} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\frac{(-2ab^2)^3}{20ab^4}$ $-\frac{2a^2b^2}{5}$
2. $\frac{4x^2 - 12x + 9}{9 - 6x}$ $\frac{3 - 2x}{3}$
3. $\frac{x^2 + x - 6}{x^2 - 6x - 27}$ $\frac{x - 2}{x - 9}$
4. $\frac{3m^3 - 3m}{6m^4} \cdot \frac{4m^5}{m + 1}$ $2m^2(m - 1)$
5. $\frac{c^2 - 3c}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3}$ $\frac{c}{c - 5}$
6. $\frac{(m - 3)^2}{m^2 - 6m + 9} \cdot \frac{m^3 - 9m}{m^2 - 9}$ m
7. $\frac{6xy^4}{25z^3} \div \frac{18xz^2}{5y}$ $\frac{y^5}{15z^5}$
8. $\frac{16p^2 - 8p + 1}{14p^4} \div \frac{4p^2 + 7p - 2}{7p^5}$ $\frac{p(4p - 1)}{2(p + 2)}$
9. $\frac{2m - 1}{m^2 - 3m - 10} \div \frac{4m^2 - 1}{4m + 8}$ $\frac{4}{(2m + 1)(m - 5)}$

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NAME ______________________ DATE __________ PERIOD _____

8-1 Study Guide and Intervention *(continued)*

Multiplying and Dividing Rational Expressions

Simplify Complex Fractions A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example Simplify $\frac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}}$.

$$\frac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}} = \frac{3s - 1}{s} \div \frac{3s^2 + 8s - 3}{s^4}$$ Express as a division problem.

$$= \frac{3s - 1}{s} \cdot \frac{s^4}{3s^2 + 8s - 3}$$ Multiply by the reciprocal of the divisor.

$$= \frac{\cancel{(3s - 1)}\cancel{s^4}}{\cancel{s}\cancel{(3s - 1)}(s + 3)}$$ Factor.

$$= \frac{s^3}{s + 3}$$ Simplify.

Exercises

Simplify.

1. $\frac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$ $\frac{xyz}{a^5}$
2. $\frac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$ $\frac{ac^7}{by}$
3. $\frac{\frac{b^2 - 1}{3b + 2}}{\frac{b + 1}{3b^2 - b - 2}}$ $(b - 1)^2$
4. $\frac{\frac{b^2 - 100}{b^2}}{\frac{3b^2 - 31b + 10}{2b}}$ $\frac{2(b + 10)}{b(3b - 1)}$
5. $\frac{\frac{x - 4}{x^2 + 6x + 9}}{\frac{x^2 - 2x - 8}{3 + x}}$ $\frac{1}{(x + 3)(x + 2)}$
6. $\frac{\frac{a^2 - 16}{a + 2}}{\frac{a^2 + 3a - 4}{a^2 + a - 2}}$ $a - 4$
7. $\frac{\frac{2x^2 + 9x + 9}{x + 1}}{\frac{10x^2 + 19x + 6}{5x^2 + 7x + 2}}$ $x + 3$
8. $\frac{\frac{b + 2}{b^2 - 6b + 8}}{\frac{b^2 + b - 2}{b^2 - 16}}$ $\frac{b + 4}{(b - 1)(b - 2)}$
9. $\frac{\frac{x^2 - x - 2}{x^3 + 6x^2 - x - 30}}{\frac{x + 1}{x + 3}}$ $\frac{1}{x + 5}$

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NAME ______________________ DATE __________ PERIOD _____

8-1 Skills Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{21x^3y}{14x^2y^2}$ $\frac{3x}{2y}$

2. $\frac{5ab^3}{25a^2b^2}$ $\frac{b}{5a}$

3. $\frac{(x^6)^3}{(x^3)^4}$ x^6

4. $\frac{8y^2(y^6)^3}{4y^{24}}$ $\frac{2}{y^4}$

5. $\frac{18}{2x-6}$ $\frac{9}{x-3}$

6. $\frac{x^2-4}{(x-2)(x+1)}$ $\frac{x+2}{x+1}$

7. $\frac{3a^2-24a}{3a^2+12a}$ $\frac{a-8}{a+4}$

8. $\frac{3m}{2n}\cdot\frac{n^3}{6}$ $\frac{mn^2}{4}$

9. $\frac{24e^3}{5f^2}\cdot\frac{10(ef)^3}{8e^5f}$ $6e$

10. $\frac{5s^2}{s^2-4}\cdot\frac{s+2}{10s^5}$ $\frac{1}{2s^3(s-2)}$

11. $\frac{7g}{y^2}\div 21g^3$ $\frac{1}{3g^2y^2}$

12. $\frac{80y^4}{49z^5v^7}\div\frac{25y^5}{14z^{12}v^5}$ $\frac{32z^7}{35v^2y}$

13. $\frac{3x^2}{x+2}\div\frac{3x}{x^2-4}$ $x(x-2)$

14. $\frac{q^2+2q}{6q}\div\frac{q^2-4}{3q^2}$ $\frac{q^2}{2(q-2)}$

15. $\frac{w^2-5w-24}{w+1}\cdot\frac{w^2-6w-7}{w+3}$ $(w-8)(w-7)$

16. $\frac{t^2+19t+84}{4t-4}\cdot\frac{2t-2}{t^2+9t+14}$ $\frac{t+12}{2(t+2)}$

17. $\frac{x^2-5x+4}{2x-8}\div(3x^2-3x)$ $\frac{1}{6x}$

18. $\frac{16a^2+40a+25}{3a^2-10a-8}\div\frac{4a+5}{a^2-8a+16}$ $\frac{(4a+5)(a-4)}{3a+2}$

19. $\frac{\frac{c^2}{2d^2}}{-\frac{c^6}{5d}}$ $-\frac{5}{2c^4d}$

20. $\frac{\frac{a^2-b^2}{4a}}{\frac{a+b}{2a}}$ $\frac{a-b}{2}$

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NAME ______________________ DATE __________ PERIOD _____

8-1 Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{9a^2b^3}{27a^4b^4c}$ $\frac{1}{3a^2bc}$

2. $\frac{(2m^3n^2)^3}{-18m^5n^4}$ $-\frac{4m^4n^2}{9}$

3. $\frac{10y^2+15y}{35y^2-5y}$ $\frac{2y+3}{7y-1}$

4. $\frac{2k^2-k-15}{k^2-9}$ $\frac{2k+5}{k+3}$

5. $\frac{25-v^2}{3v^2-13v-10}$ $-\frac{v+5}{3v+2}$

6. $\frac{x^4+x^3-2x^2}{x^4-x^3}$ $\frac{x+2}{x}$

7. $\frac{-2u^3y}{15xz^5}\cdot\frac{25x^3}{14u^2y^2}$ $-\frac{5ux^2}{21yz^5}$

8. $\frac{a+y}{6}\cdot\frac{4}{y+a}$ $\frac{2}{3}$

9. $\frac{n^5}{n-6}\cdot\frac{n^2-6n}{n^8}$ $\frac{1}{n^2}$

10. $\frac{a-y}{w+n}\cdot\frac{w^2-n^2}{y-a}$ $n-w$

11. $\frac{x^2-5x-24}{6x+2x^2}\cdot\frac{5x^2}{8-x}$ $-\frac{5x}{2}$

12. $\frac{x-5}{10x-2}\cdot\frac{25x^2-1}{x^2-10x+25}$ $\frac{5x+1}{2(x-5)}$

13. $\frac{a^5y^3}{wy^7}\div\frac{a^3w^2}{w^5y^2}$ $\frac{a^2w^2}{y^2}$

14. $\left(\frac{2xy}{w^2}\right)^3\div\frac{24x^2}{w^5}$ $\frac{xy^3}{3w}$

15. $\frac{x+y}{6}\div\frac{x^2-y^2}{3}$ $\frac{1}{2(x-y)}$

16. $\frac{3x+6}{x^2-9}\div\frac{6x^2+12x}{4x+12}$ $\frac{2}{x(x-3)}$

17. $\frac{2s^2-7s-15}{(s+4)^2}\div\frac{s^2-10s+25}{s+4}$ $\frac{2s+3}{(s+4)(s-5)}$

18. $\frac{9-a^2}{a^2+5a+6}\div\frac{2a-6}{5a+10}$ $-\frac{5}{2}$

19. $\frac{\frac{2x+1}{x}}{\frac{4-x}{x}}$ $\frac{2x+1}{4-x}$

20. $\frac{\frac{x^2-9}{4}}{\frac{3-x}{8}}$ $-2(x+3)$

21. $\frac{\frac{x^3+2^3}{x^2-2x}}{\frac{(x+2)^3}{x^2+4x+4}}$ $\frac{x^2-2x+4}{x(x-2)}$

22. **GEOMETRY** A right triangle with an area of x^2-4 square units has a leg that measures $2x+4$ units. Determine the length of the other leg of the triangle.
$x-2$ units

23. **GEOMETRY** A rectangular pyramid has a base area of $\frac{x^2+3x-10}{2x}$ square centimeters and a height of $\frac{x^2-3x}{x^2-5x+6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid. $\frac{x+5}{6}$ cm^3

NAME ______________________ DATE ____________ PERIOD _____

8-1 Word Problem Practice

Multiplying and Dividing Rational Expressions

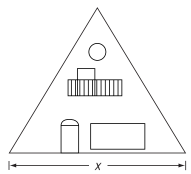
1. JELLY BEANS A large vat contains G green jelly beans and R red jelly beans. A bag of 100 red and 100 green jelly beans is added to the vat. What is the new ratio of red to green jelly beans in the vat?

$\frac{R + 100}{G + 100}$

2. MILEAGE Beth's car gets 15 miles per gallon in the city and 26 miles per gallon on the highway. Beth uses C gallons of gas in the city and H gallons of gas on the highway. Write an expression for the average number of miles per gallon that Beth gets with her car in terms of C and H.

$\frac{15C + 26H}{C + H}$

3. HEIGHT The front face of a Nordic house is triangular. The surface area of the face is $x^2 + 3x + 10$ where x is the base of the triangle.



What is the height of the triangle in terms of x?

$h = \frac{2x^2 + 6x + 20}{x}$

4. OIL SLICKS David was moving a drum of oil around his circular outdoor pool when the drum cracked, and oil spilled into the pool. The oil spread itself evenly over the surface of the pool. Let V denote the volume of oil spilled and let r be the radius of the pool. Write an equation for the thickness of the oil layer.

$h = \frac{V}{\pi r^2}$

RUNNING For Exercises 5 and 6, use the following information.

Harold runs to the local food mart to buy a gallon of soy milk. Because he is weighed down on his return trip, he runs slower on the way back. He travels S_1 feet per second on the way to the food mart and S_2 feet per second on the way back. Let d be the distance he has to run to get to the food mart. Remember: distance = rate × time.

5. Write an equation that gives the total time Harold spent running for this errand.

$t = \frac{d}{S_1} + \frac{d}{S_2}$

6. What speed would Harold have to run if he wanted to maintain a constant speed for the entire trip yet take the same amount of time running?

$\frac{2S_1S_2}{S_1 + S_2}$

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NAME ______________________ DATE ____________ PERIOD _____

8-1 Enrichment

Dimensional Analysis

Scientists always express the units of measurement in their solution. It is insufficient and ambiguous to state a solution regarding distance as 17; Seventeen what, feet, miles, meters? Often it is helpful to analyze the units of the quantities in a formula to determine the desired units of an output. For example, it is known that torque is the product of force and distance, but what are the units of force?

The units also depend on the measuring system. The two most commonly used systems are the British system and the international system of units (SI). Some common units of the British system are inches, feet, miles, and pounds. Common SI units include meters, kilometers, Newtons, and grams. Frequently conversion from one system to another is necessary and accomplished by multiplication by conversion factors.

Consider changing units from miles per hour to kilometers per hour. What is 60 miles per hour in kilometers per hour? Use the conversion 1 ft = 30.5 cm.

$$60\,\frac{\text{mi}}{\text{h}} = 60\,\frac{\text{mi}}{\text{h}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{30.5\text{ cm}}{1\text{ ft}} \times \frac{1\text{ m}}{100\text{ cm}} \times \frac{1\text{ km}}{1000\text{ m}} = 96.62\,\frac{\text{km}}{\text{h}}$$

1. The SI unit for force is a Newton (N) and the SI unit for distance is meters or centimeters. The British unit for force is pounds and the British unit for distance is feet or inches. Using the formula for torque (Torque = Force times Distance), determine the SI unit and the British unit for torque.

Possible answers are $N \cdot m$ and $ft \cdot lb$ or $N \cdot cm$ and $inch \cdot lb$

2. The density of a fluid is given by the formula $density = \frac{mass}{volume}$. Suppose that a volume of a fluid in a cylindrical can is $\pi r^2 h$, where r and h are measured in meters. Find an expression for the mass, given in kilograms (kg), of gasoline, which has a known density of $680\,\frac{kg}{m^3}$.

$680\pi r^2 h$ **kg**

3. Convert the following measurements.

a. 72 miles/hour to feet/second

105.6 feet/second

b. 32 pounds/square inch to pounds per square foot

4608 pounds per square foot

c. 100 kilometers/hour to miles per hour

62.1 miles per hour

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NAME ______________________ DATE __________ PERIOD _____

8-2 Lesson Reading Guide

Adding and Subtracting Rational Expressions

Get Ready for the Lesson

Read the introduction to Lesson 8-2 in your textbook.

A person is standing 5 feet from a camera that has a lens with a focal length of 3 feet. Write an equation that you could solve to find how far the film should be from the lens to get a perfectly focused photograph.

$\frac{1}{q} = \frac{1}{3} - \frac{1}{5}$

Read the Lesson

1. a. In work with rational expressions, LCD stands for **least common denominator** and LCM stands for **least common multiple**. The LCD is the **LCM** of the denominators.

b. To find the LCM of two or more numbers or polynomials, **factor** each number or **polynomial**. The LCM contains each **factor** the **greatest** number of times it appears as a **factor**.

2. To add $\frac{x^2 - 3}{x^2 - 5x + 6}$ and $\frac{x - 4}{x^3 - 4x^2 + 4x}$, you should first factor the **denominator** of each fraction. Then use the factorizations to find the **LCM** of $x^2 - 5x + 6$ and $x^3 - 4x^2 + 4x$. This is the **LCD** for the two fractions.

3. When you add or subtract fractions, you often need to rewrite the fractions as equivalent fractions. You do this so that the resulting equivalent fractions will each have a denominator equal to the **LCD** of the original fractions.

4. To add or subtract two fractions that have the same denominator, you add or subtract their **numerators** and keep the same **denominator**.

5. The sum or difference of two rational expressions should be written as a polynomial or as a fraction in **simplest form**.

Remember What You Learned

6. Some students have trouble remembering whether a common denominator is needed to add and subtract rational expressions or to multiply and divide them. How can your knowledge of working with fractions in arithmetic help you remember this?

Sample answer: In arithmetic, a common denominator is needed to add and subtract fractions, but not to multiply and divide them. The situation is the same for rational expressions.

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NAME ______________________ DATE __________ PERIOD _____

8-2 Study Guide and Intervention

Adding and Subtracting Rational Expressions

LCM of Polynomials To find the least common multiple of two or more polynomials, factor each expression. The LCM contains each factor the greatest number of times it appears as a factor.

Example **Find the LCM of $16p^2q^3r$, $40pq^4r^2$, and $15p^3r^4$.**

$16p^2q^3r = 2^4 \cdot p^2 \cdot q^3 \cdot r$
$40pq^4r^2 = 2^3 \cdot 5 \cdot p \cdot q^4 \cdot r^2$
$15p^3r^4 = 3 \cdot 5 \cdot p^3 \cdot r^4$
$\text{LCM} = 2^4 \cdot 3 \cdot 5 \cdot p^3 \cdot q^4 \cdot r^4$
$= 240p^3q^4r^4$

Example **Find the LCM of $3m^2 - 3m - 6$ and $4m^2 + 12m - 40$.**

$3m^2 - 3m - 6 = 3(m + 1)(m - 2)$
$4m^2 + 12m - 40 = 4(m - 2)(m + 5)$
$\text{LCM} = 12(m + 1)(m - 2)(m + 5)$

Exercises

Find the LCM of each set of polynomials.

1. $14ab^2, 42bc^3, 18a^2c$ **$126a^2b^2c^3$**

2. $8cdf^3, 28c^2f, 35d^4f^2$ **$280c^2d^4f^3$**

3. $65x^4y, 10x^2y^2, 26y^4$ **$130x^4y^4$**

4. $11mn^5, 18m^2n^3, 20mn^4$ **$1980m^2n^5$**

5. $15a^4b, 50a^2b^2, 40b^8$ **$600a^4b^8$**

6. $24p^7q, 30p^2q^2, 45pq^3$ **$360p^7q^3$**

7. $39b^2c^2, 52b^4c, 12c^3$ **$156b^4c^3$**

8. $12xy^4, 42x^2y, 30x^2y^3$ **$420x^2y^4$**

9. $56stv^2, 24s^2v^2, 70t^3v^3$ **$840s^2t^3v^3$**

10. $x^2 + 3x, 10x^2 + 25x - 15$ **$5x(x + 3)(2x - 1)$**

11. $9x^2 - 12x + 4, 3x^2 + 10x - 8$ **$(3x - 2)^2(x + 4)$**

12. $22x^2 + 66x - 220, 4x^2 - 16$ **$44(x - 2)(x + 2)(x + 5)$**

13. $8x^2 - 36x - 20, 2x^2 + 2x - 60$ **$4(x - 5)(x + 6)(2x + 1)$**

14. $5x^2 - 125, 5x^2 + 24x - 5$ **$5(x - 5)(x + 5)(5x - 1)$**

15. $3x^2 - 18x + 27, 2x^3 - 4x^2 - 6x$ **$6x(x - 3)^2(x + 1)$**

16. $45x^2 - 6x - 3, 45x^2 - 5$ **$15(5x + 1)(3x - 1)(3x + 1)$**

17. $x^3 + 4x^2 - x - 4, x^2 + 2x - 3$ **$(x - 1)(x + 1)(x + 3)(x + 4)$**

18. $54x^3 - 24x, 12x^2 - 26x + 12$ **$6x(3x + 2)(3x - 2)(2x - 3)$**

NAME ______________________ DATE ____________ PERIOD _____

8-2 Study Guide and Intervention *(continued)*

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions To add or subtract rational expressions, follow these steps.

Step 1 If necessary, find equivalent fractions that have the same denominator.
Step 2 Add or subtract the numerators.
Step 3 Combine any like terms in the numerator.
Step 4 Factor if possible.
Step 5 Simplify if possible.

Example **Simplify** $\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$.

$\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$

$= \frac{6}{2(x+3)(x-2)} - \frac{2}{(x-2)(x+2)}$ Factor the denominators.

$= \frac{6(x+2)}{2(x+3)(x-2)(x+2)} - \frac{2 \cdot 2(x+3)}{2(x+3)(x-2)(x+2)}$ The LCD is $2(x+3)(x-2)(x+2)$.

$= \frac{6(x+2) - 4(x+3)}{2(x+3)(x-2)(x+2)}$ Subtract the numerators.

$= \frac{6x + 12 - 4x - 12}{2(x+3)(x-2)(x+2)}$ Distributive Property

$= \frac{2x}{2(x+3)(x-2)(x+2)}$ Combine like terms.

$= \frac{x}{(x+3)(x-2)(x+2)}$ Simplify.

Exercises

Simplify each expression.

1. $\frac{-7xy}{3x} + \frac{4y^2}{2y}$ $-\frac{y}{3}$

2. $\frac{2}{x-3} - \frac{1}{x-1}$ $\frac{x+1}{(x-1)(x-3)}$

3. $\frac{4a}{3bc} - \frac{15b}{5ac}$ $\frac{4a^2 - 9b^2}{3abc}$

4. $\frac{3}{x+2} + \frac{4x+5}{3x+6}$ $\frac{4x+14}{3x+6}$

5. $\frac{3x+3}{x^2+2x+1} + \frac{x-1}{x^2-1}$ $\frac{4}{x+1}$

6. $\frac{4}{4x^2-4x+1} - \frac{5x}{20x^2-5}$ $\frac{-2x^2+9x+4}{(2x+1)(2x-1)^2}$

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8-2 Skills Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $12c, 6c^2d$ $12c^2d$

2. $18a^3bc^2, 24b^2c^2$ $72a^3b^2c^2$

3. $2x - 6, x - 3$ $2(x-3)$

4. $5a, a - 1$ $5a(a-1)$

5. $t^2 - 25, t + 5$ $(t+5)(t-5)$

6. $x^2 - 3x - 4, x + 1$ $(x-4)(x+1)$

Simplify each expression.

7. $\frac{3}{x} + \frac{5}{y}$ $\frac{5x+3y}{xy}$

8. $\frac{3}{8p^2q} + \frac{5}{4p^2q}$ $\frac{13}{8p^2q}$

9. $\frac{2c-7}{3} + 4$ $\frac{2c+5}{3}$

10. $\frac{2}{m^2n} + \frac{5}{n}$ $\frac{2+5m^2}{m^2n}$

11. $\frac{12}{5y^2} - \frac{2}{5yz}$ $\frac{12z-2y}{5y^2z}$

12. $\frac{7}{4gh} + \frac{3}{4h^2}$ $\frac{7h+3g}{4gh^2}$

13. $\frac{2}{a+2} - \frac{3}{2a}$ $\frac{a-6}{2a(a+2)}$

14. $\frac{5}{3b+d} - \frac{2}{3bd}$ $\frac{15bd-6b-2d}{3bd(3b+d)}$

15. $\frac{3}{w-3} - \frac{2}{w^2-9}$ $\frac{3w+7}{(w-3)(w+3)}$

16. $\frac{3t}{2-x} + \frac{5}{x-2}$ $\frac{5-3t}{x-2}$

17. $\frac{m}{m-n} - \frac{m}{n-m}$ $\frac{2m}{m-n}$

18. $\frac{4z}{z-4} + \frac{z+4}{z+1}$ $\frac{5z^2+4z-16}{(z-4)(z+1)}$

19. $\frac{1}{x^2+2x+1} + \frac{x}{x+1}$ $\frac{x^2+x+1}{(x+1)^2}$

20. $\frac{2x+1}{x-5} - \frac{4}{x^2-3x-10}$ $\frac{2x^2+5x-2}{(x-5)(x+2)}$

21. $\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$ $\frac{n+2}{n-3}$

22. $\frac{3}{y^2+y-12} - \frac{2}{y^2+6y+8}$ $\frac{y+12}{(y+4)(y-3)(y+2)}$

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8-2 Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. x^2y, xy^3 — x^2y^3
2. a^2b^3c, abc^4 — $a^2b^3c^4$
3. $x + 1, x + 3$ — $(x + 1)(x + 3)$
4. $g - 1, g^2 + 3g - 4$ — $(g - 1)(g + 4)$
5. $2r + 2, r^2 + r, r + 1$ — $2r(r + 1)$
6. $3, 4w + 2, 4w^2 - 1$ — $6(2w + 1)(2w - 1)$
7. $x^2 + 2x - 8, x + 4$ — $(x + 4)(x - 2)$
8. $x^2 - x - 6, x^2 + 6x + 8$ — $(x + 2)(x + 4)(x - 3)$
9. $d^2 + 6d + 9, 2(d^2 - 9)$ — $2(d - 3)(d + 3)^2$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$ — $\frac{20 - 21b}{24ab}$
11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$ — $\frac{25y^2 - 12x^2}{60x^4y^3}$
12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$ — $\frac{2d^2 + 9c}{12c^2d^3}$
13. $\frac{4m}{3mn} + 2$ — $\frac{2(2 + 3n)}{3n}$
14. $2x - 5 - \frac{x - 8}{x + 4}$ — $\frac{2(x + 3)(x - 2)}{x + 4}$
15. $\frac{4}{a - 3} + \frac{9}{a - 5}$ — $\frac{13a - 47}{(a - 3)(a - 5)}$
16. $\frac{16}{x^2 - 16} + \frac{2}{x + 4}$ — $\frac{2}{x - 4}$
17. $\frac{2 - 5m}{m - 9} + \frac{4m - 5}{9 - m}$ — $\frac{7 - 9m}{m - 9}$
18. $\frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$ — $\frac{2y - 1}{(y + 2)(y - 1)}$
19. $\frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12}$ — $\frac{5}{2(x + 2)}$
20. $\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}$ — $\frac{2p^2 - 2p + 1}{(p - 2)(p + 3)(p - 3)}$
21. $\frac{1}{5n} - \frac{3}{4} + \frac{7}{10n}$ — $\frac{3(6 - 5n)}{20n}$
22. $\frac{2a}{a - 3} - \frac{2a}{a + 3} + \frac{36}{a^2 - 9}$ — $\frac{12}{a - 3}$
23. $\frac{\frac{2}{x - y} + \frac{1}{x + y}}{\frac{1}{x - y}}$ — $\frac{3x + y}{x + y}$
24. $\frac{\frac{r + 6}{r} - \frac{1}{r + 2}}{\frac{r^2 + 4r + 3}{r^2 + 2r}}$ — $\frac{r + 4}{r + 1}$

25. **GEOMETRY** The expressions $\frac{5x}{2}$, $\frac{20}{x + 4}$, and $\frac{10}{x - 4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle. $\frac{5(x^3 - 4x - 16)}{2(x - 4)(x + 4)}$

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip.
$\frac{4r}{(r + 2)(r - 2)}$ h

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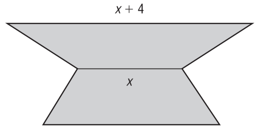
8-2 Word Problem Practice

Adding and Subtracting Rational Expressions

1. **SQUARES** Susan's favorite perfect square is s^2 and Travis' is t^2, where s and t are whole numbers. What perfect square is guaranteed to be divisible by both Susan's and Travis' favorite perfect squares regardless of their specific value?
s^2t^2

2. **ELECTRIC POTENTIAL** The electrical potential function between two electrons is given by a formula that has the form $\frac{1}{r} + \frac{1}{1 - r}$. Simplify this expression.
$\frac{1}{r(1 - r)}$

3. **TRAPEZOIDS** The cross section of a stand consists of two trapezoids stacked one on top of the other.



The total area of the cross section is x^2 square units. Assuming the trapezoids have the same height, write an expression for the height of the stand in terms of x. Put your answer in simplest form. (Recall that the area of a trapezoid with height h and bases b_1 and b_2 is given by $\frac{1}{2}h(b_1 + b_2)$.)
$\frac{x^2}{2x + 3}$

4. **FRACTIONS** In the seventeenth century, Lord Brouncker wrote down a most peculiar mathematical equation:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{\ddots}}}}$$

This is an example of a continued fraction. Simplify the continued fraction $n + \cfrac{1}{n + \cfrac{1}{n}}$.
$\frac{n^3 + 2n}{n^2 + 1}$

RELAY RACE For Exercises 5-7, use the following information.

Mark, Connell, Zack, and Moses run the 4 by 400 meter relay together. Their average speeds were s, $s + 0.5$, $s - 0.5$, and $s + 1$ meters per second, respectively.

5. What were their individual times for their own legs of the race?
$\frac{400}{s}, \frac{400}{s + \frac{1}{2}}, \frac{400}{s - \frac{1}{2}}, \frac{400}{s - 1}$

6. Write an expression for their time as a team. Write your answer as a ratio of two polynomials.
$400\frac{16s^3 - 12s^2 - 2s + 1}{4s^4 - 4s^3 - s^2 + s}$

7. If s was 6 meters per second, what was the team's time? Round your answer to the nearest second.
281 seconds

Lesson 8-2

Answers

NAME ______________________ DATE ____________ PERIOD _____

8-2 Enrichment

Zeno's Paradox

The Greek philosopher Zeno of Elea (born sometime between 495 and 480 B.C.) proposed four paradoxes to challenge the notions of space and time. Zeno's first paradox works like this:

Suppose you are on your way to school. Assume you are able to cover half of the remaining distance each minute that you walk. You leave your house at 7:45 A.M. After the first minute, you are half of the way to school. In the next minute you cover half of the remaining distance to school, and at 7:47 A.M. you are three-quarters of the way to school. This pattern continues each minute. At what time will you arrive at school? Before 8:00 A.M.? Before lunch?

Since space is infinitely divisible, we can repeat this pattern forever. Thus, on the way to school you must reach an infinite number of 'midpoints' in a finite time. This is impossible, so you can never reach your goal. In general, according to Zeno anyone who wants to move from one point to another must meet these requirements, and motion is impossible. Therefore, what we perceive as motion is merely an illusion.

Addition of fractions can be defined by $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, similarly for subtraction.

Assume your house is one mile from school. At 7:46 A.M., you have walked half of a mile, so you have left $1 - \frac{1}{2}$, or $\frac{1}{2}$ a mile. At 7:47 A.M. you only have $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ of a mile to go.

To determine how far you have walked and how far away from the school you are at 7:48 A.M., add the distances walked each minute, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$ of a mile so far and you still have $1 - \frac{7}{8} = \frac{1}{8}$ of a mile to go.

1. Determine how far you have walked and how far away from the school you are at 7:50 A.M.
   You have walked $\frac{31}{32}$ of a mile, and will be $\frac{1}{32}$ of a mile away from school.

2. Suppose instead of covering one-half the distance to school each minute, you cover three-quarters of the distance remaining to school each minute, now will you be able to make it to school on time? Determine how far you still have left to go at 7:47 A.M.
   No. You will have $\frac{1}{16}$ of a mile remaining at 7:47 A.M.

3. Suppose that instead of covering one-half or three-quarters of the distance to school each minute, you cover $\frac{1}{x + 1}$ of the distance remaining, where x is a whole number greater than 2. What is your distance from school at 7:46 A.M.?
   You are $\frac{x^2}{(x + 1)^2}$ of a mile from school at 7:46 A.M.

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8-3 Lesson Reading Guide

Graphing Rational Functions

Get Ready for the Lesson

Read the introduction to Lesson 8-3 in your textbook.

- If 15 students contribute to the gift, how much would each of them pay? **$10**
- If each student pays $5, how many students contributed? **30 students**

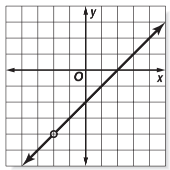
Read the Lesson

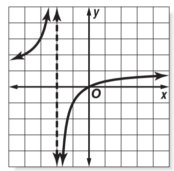
1. Which of the following are rational functions? **A and C**

   A. $f(x) = \frac{1}{x - 5}$ **B.** $g(x) = \sqrt{x}$ **C.** $h(x) = \frac{x^2 - 25}{x^2 + 6x + 9}$

2. **a.** Graphs of rational functions may have breaks in **continuity**. These may occur as vertical **asymptotes** or as point **discontinuities**. The **domain** of a rational function is limited to values for which the function is defined.

   b. The graphs of two rational functions are shown below.

   I.
   

   II.
   

   Graph I has a **point discontinuity** at $x =$ **−2**.

   Graph II has a **vertical asymptote** at $x =$ **−2**.

   Match each function with its graph above.

   $f(x) = \frac{x}{x + 2}$ **II** $g(x) = \frac{x^2 - 4}{x + 2}$ **I**

Remember What You Learned

3. One way to remember something new is to see how it is related to something you already know. How can knowing that division by zero is undefined help you to remember how to find the places where a rational function has a point discontinuity or an asymptote?
   Sample answer: A point discontinuity or vertical asymptote occurs where the function is undefined, that is, where the denominator of the related rational expression is equal to 0. Therefore, set the denominator equal to zero and solve for the variable.

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8-3 Study Guide and Intervention

Graphing Rational Functions

Domain and Range

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Domain	The domain of a rational function is limited to values for which the function is defined.
Vertical Asymptote	An asymptote is a line that the graph of a function approaches. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.
Horizontal Asymptote	Often a horizontal asymptote occurs in the graph of a rational function where a value is excluded from the range.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$ **asymptotes: $x = 2$, $x = -5$**

2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$ **hole: $x = \frac{5}{2}$**

3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$ **asymptote: $x = 0$; hole $x = 4$**

4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$ **asymptote: $x = -2$; hole: $x = \frac{1}{3}$**

5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$ **asymptotes: $x = 1$, $x = -7$**

6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$ **asymptote: $x = -3$**

7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$ **asymptotes: $x = 1$, $x = 5$**

8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$ **asymptote: $x = -3$; hole: $x = \frac{3}{2}$**

9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$ **holes: $x = 1$, $x = 3$**

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8-3 Study Guide and Intervention *(continued)*

Graphing Rational Functions

Graph Rational Functions

Use the following steps to graph a rational function.

Step 1 First see if the function has any vertical asymptotes or point discontinuities.
Step 2 Draw any vertical asymptotes.
Step 3 Make a table of values.
Step 4 Plot the points and draw the graph.

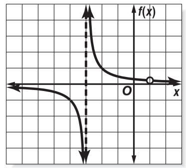
Example **Graph $f(x) = \frac{x - 1}{x^2 + 2x - 3}$.**

$\frac{x - 1}{x^2 + 2x - 3} = \frac{x - 1}{(x - 1)(x + 3)}$ or $\frac{1}{x + 3}$

Therefore the graph of $f(x)$ has an asymptote at $x = -3$ and a point discontinuity at $x = 1$.

Make a table of values. Plot the points and draw the graph.

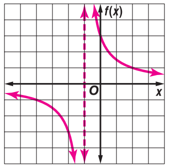
x	−2.5	−2	−1	−3.5	−4	−5
$f(x)$	2	1	0.5	−2	−1	−0.5



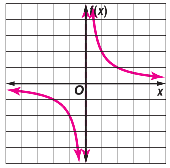
Exercises

Graph each rational function.

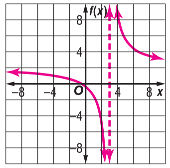
1. $f(x) = \frac{3}{x + 1}$



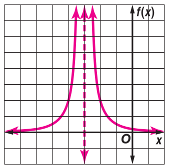
2. $f(x) = \frac{2}{x}$



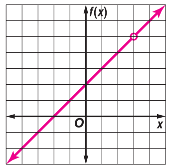
3. $f(x) = \frac{2x + 1}{x - 3}$



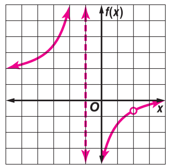
4. $f(x) = \frac{2}{(x + 3)^2}$



5. $f(x) = \frac{x^2 - x - 6}{x - 3}$



6. $f(x) = \frac{x^2 - 6x + 8}{x^2 - x - 2}$



Lesson 8-3

Answers

NAME ______________________ DATE ____________ PERIOD _____

8-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{3}{x^2 - 2x - 8}$ **asymptotes: $x = 4$, $x = -2$**

2. $f(x) = \frac{10}{x^2 - 13x + 36}$ **asymptotes: $x = 4$, $x = 9$**

3. $f(x) = \frac{x + 12}{x^2 + 10x - 24}$ **asymptote: $x = 2$; hole: $x = -12$**

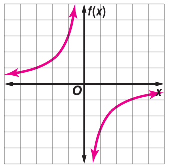
4. $f(x) = \frac{x - 1}{x^2 - 4x + 3}$ **asymptote: $x = 3$; hole: $x = 1$**

5. $f(x) = \frac{x^2 + 8x + 12}{x + 2}$ **hole: $x = -2$**

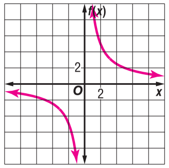
6. $f(x) = \frac{x^2 + x - 12}{x - 3}$ **hole: $x = 3$**

Graph each rational function.

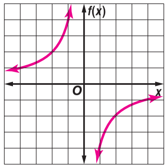
7. $f(x) = \frac{-3}{x}$



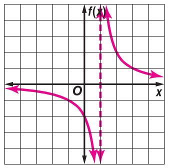
8. $f(x) = \frac{10}{x}$



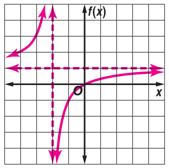
9. $f(x) = \frac{-4}{x}$



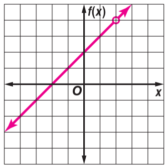
10. $f(x) = \frac{2}{x - 1}$



11. $f(x) = \frac{x}{x + 2}$



12. $f(x) = \frac{x^2 - 4}{x - 2}$



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NAME ______________________ DATE ____________ PERIOD _____

8-3 Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$ **asymptotes: $x = 2$, $x = -5$**

2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$ **asymptote: $x = 3$; hole: $x = 7$**

3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$ **asymptote: $x = -2$**

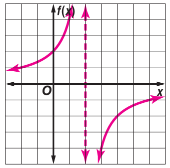
4. $f(x) = \frac{x^2 - 100}{x + 10}$ **hole: $x = -10$**

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$ **hole: $x = 6$**

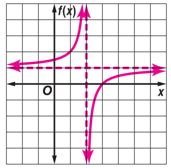
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$ **hole: $x = -5$**

Graph each rational function.

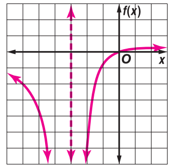
7. $f(x) = \frac{-4}{x - 2}$



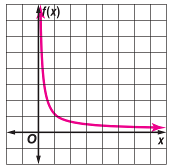
8. $f(x) = \frac{x - 3}{x - 2}$



9. $f(x) = \frac{3x}{(x + 3)^2}$

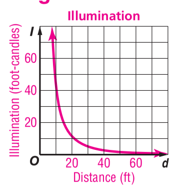


10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x > 0, y > 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour? What domain and range values are meaningful in the context of the problem?



$\frac{5}{12}$; Sample answer: The number of hours it takes her father to give the shed a coat of paint should be positive. Therefore, only values of x greater than 0 and values of $f(x)$ greater than $\frac{1}{16}$ are meaningful.

11. **LIGHT** The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 < I \leq 80$ and $0 < d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source? What domain and range values are meaningful in the context of the problem? **11.25 foot-candles; Sample answer: The distance of the object from the source should be positive. Therefore, only values of d greater than 0 and values of $I(d)$ greater than 0 are meaningful.**



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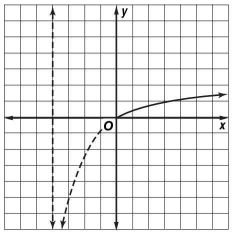
8-3 Word Problem Practice

Graphing Rational Expressions

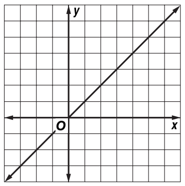
1. **ROAD TRIP** Robert and Sarah start off on a road trip from the same house. During the trip, Robert's and Sarah's cars remain separated by a constant distance. The graph shows the ratio of the distance Sarah has traveled to the distance Robert has traveled. The dotted line shows how this graph would be extended to hypothetical negative values of x. What does the x-coordinate of the vertical asymptote represent?



the distance by which Sarah trails Robert

2. **GRAPHS** Alma graphed the function

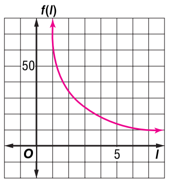
$f(x) = \frac{x^2 - 4x}{x - 4}$ below.



There is a problem with her graph. Explain how to correct it.

The point (4, 4) needs to be erased and a small circle put around it.

3. **FINANCE** A quick way to get an idea of how many years before a savings account will double at an interest rate of I percent compounded annually, is to divide I into 72. Sketch a graph of the function $f(I) = \frac{72}{I}$.



4. **NEWTON** Sir Isaac Newton studied the rational function

$f(x) = \frac{ax^3 + bx^2 + cx + d}{x}$.

Assuming that $d \neq 0$, where will there be a vertical asymptote to the graph of this function?

$x = 0$

BATTING AVERAGES For Exercises 5 and 6, use the following information.

Josh has made 26 hits in 80 at bats for a batting average of .325. Josh goes on a hitting streak and makes x hits in the next $2x$ at bats.

5. What function describes Josh's batting average during this streak?

$f(x) = \frac{26 + x}{80 + 2x}$

6. What is the equation of the horizontal asymptote to the graph of the function you wrote for Exercise 5? What is its meaning?

$y = 0.5$; 0.5 represents an upper bound on Josh's batting average if his hit rate does not change

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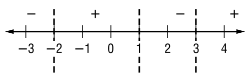
8-3 Enrichment

Characteristics of Rational Function Graphs

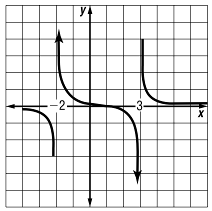
Use the information in the table to graph rational functions

CHARACTERISTIC	MEANING	HOW TO FIND IT
Vertical asymptotes	A vertical line at an x value where the rational function is undefined	Set the denominator equal to zero and solve for x.
Horizontal asymptotes	A horizontal line that the rational function	Study the end-behaviors.
Right end-behavior	How the graph behaves at large positive values of x	Evaluate the rational expression at increasing positive values of x.
Left end-behavior	How the graph behaves at large negative values of x	Evaluate the rational expression at increasing negative values of x.
Roots, zeros, or x-intercepts	Point(s) where the graph crosses the x-axis	Set the numerator equal to zero and solve for x.
y-intercepts	Point where the graph crosses the y-axis	Set $x = 0$ to determine the y-intercept.

Example A **sign chart** uses an x value from the left and right of each critical value to determine if the graph is positive or negative on that interval. A sign chart for $y = \frac{x - 1}{x^2 - x - 6}$ is shown below.

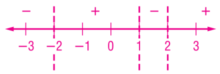


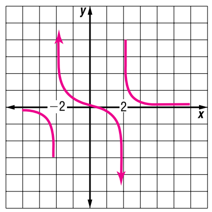
The graph of $\frac{x - 1}{x^2 - x - 6}$ is shown to the right.



Exercise

Create a sign chart for $y = \frac{x - 1}{x^2 - 4}$. Use an x-value from the left and right of each critical value to determine if the graph is positive or negative on that interval. Then graph the function.





NAME ____________________ DATE ____________ PERIOD _____

8-3 Graphing Calculator Activity

Horizontal Asymptotes and Tables

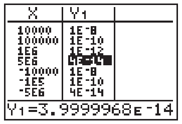
The line $y = b$ is a horizontal asymptote for the rational function $f(x)$ if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$. The horizontal asymptote can be found by using the **TABLE** feature of the graphing calculator.

Example **Find the horizontal asymptote for each function.**

a. $f(x) = \dfrac{1}{x^2 + 4x - 5}$

Enter the function into **Y1**. Place **[TblSet]** in the Ask mode. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x-list.

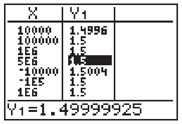
Keystrokes: [Y=] 1 [÷] [(] [X,T,θ,n] [x^2] [+] 4 [X,T,θ,n] [–] 5 [)] [2nd] [TBLSET] [▼] [▼] [▶] [ENTER] [2nd] [TABLE]. Then enter the values for x.



Notice that as x increases, y approaches 0. Thus, $y = 0$ is the horizontal asymptote.

b. $f(x) = \dfrac{3x^2}{2x^2 + 5x - 6}$

Enter the equation into **Y1**. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x-list. Note the pattern. As x increases, y approaches 1.5. Thus, $y = 1.5$ is the horizontal asymptote.



Exercises

Find the horizontal asymptote for each function.

1. $f(x) = \dfrac{2x}{x+1}$ **$y = 2$**

2. $f(x) = \dfrac{x^2 - 1}{2x^2 - 7x + 12}$ **$y = \dfrac{1}{2}$**

3. $f(x) = \dfrac{6x^3}{2x^3 - 2x^2 - 2}$ **$y = 3$**

4. $f(x) = \dfrac{2x}{3x^2 - 5x - 1}$ **$y = 0$**

5. $f(x) = \dfrac{15x^2 - 3x + 7}{x^3}$ **$y = 0$**

6. $f(x) = \dfrac{x^3 - 8x^2 - 4x + 11}{x^4 - 3x^3 - 4x - 6}$ **$y = 0$**

7. $f(x) = \dfrac{5x^2 - 3}{x - 2}$ **none**

8. $f(x) = \dfrac{6x^3}{2x^2 - 3x + 6}$ **none**

9. $f(x) = \dfrac{2x - 4}{2}$ **none**

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NAME ____________________ DATE ____________ PERIOD _____

8-4 Lesson Reading Guide

Direct, Joint, and Inverse Variation

Get Ready for the Lesson

Read the introduction to Lesson 8-4 in your textbook.

- For each additional student who enrolls in a public college, the total high-tech spending will **increase** (increase/decrease) by **$203**.
- For each decrease in enrollment of 100 students in a public college, the total high-tech spending will **decrease** (increase/decrease) by **$20,300**.

Read the Lesson

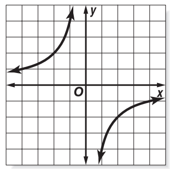
1. Write an equation to represent each of the following variation statements. Use k as the constant of variation.

a. m varies inversely as n. **$m = \dfrac{k}{n}$**

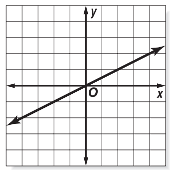
b. s varies directly as r. **$s = kr$**

c. t varies jointly as p and q. **$t = kpq$**

2. Which type of variation, direct or inverse, is represented by each graph?

a. 

inverse

b. 

direct

Remember What You Learned

3. How can your knowledge of the equation of the slope-intercept form of the equation of a line help you remember the equation for direct variation?

Sample answer: The graph of an equation expressing direct variation is a line. The slope-intercept form of the equation of a line is $y = mx + b$. In direct variation, if one of the quantities is 0, the other quantity is also 0, so $b = 0$ and the line goes through the origin. The equation of a line through the origin is $y = mx$, where m is the slope. This is the same as the equation for direct variation with $k = m$.

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8-4 Study Guide and Intervention

Direct, Joint, and Inverse Variation

Direct Variation and Joint Variation

Direct Variation	y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.
Joint Variation	y varies jointly as x and z if there is some number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.

Example **Find each value.**

a. If y varies directly as x and $y = 16$ when $x = 4$, find x when $y = 20$.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ Direct proportion

$\frac{16}{4} = \frac{20}{x_2}$ $y_1 = 16$, $x_1 = 4$, and $y_2 = 20$

$16x_2 = (20)(4)$ Cross multiply.

$x_2 = 5$ Simplify.

The value of x is 5 when y is 20.

b. If y varies jointly as x and z and $y = 10$ when $x = 2$ and $z = 4$, find y when $x = 4$ and $z = 3$.

$\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$ Joint variation

$\frac{10}{2 \cdot 4} = \frac{y_2}{4 \cdot 3}$ $y_1 = 10$, $x_1 = 2$, $z_1 = 4$, $x_2 = 4$, and $z_2 = 3$

$120 = 8y_2$ Simplify.

$y_2 = 15$ Divide each side by 8.

The value of y is 15 when $x = 4$ and $z = 3$.

Exercises

Find each value.

1. If y varies directly as x and $y = 9$ when $x = 6$, find y when $x = 8$. **12**
2. If y varies directly as x and $y = 16$ when $x = 36$, find y when $x = 54$. **24**
3. If y varies directly as x and $x = 15$ when $y = 5$, find x when $y = 9$. **27**
4. If y varies directly as x and $x = 33$ when $y = 22$, find x when $y = 32$. **48**
5. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 3$, if $y = 18$ when $x = 3$ and $z = 2$. **45**
6. Suppose y varies jointly as x and z. Find y when $x = 6$ and $z = 8$, if $y = 6$ when $x = 4$ and $z = 2$. **36**
7. Suppose y varies jointly as x and z. Find y when $x = 4$ and $z = 11$, if $y = 60$ when $x = 3$ and $z = 5$. **176**
8. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 2$, if $y = 84$ when $x = 4$ and $z = 7$. **30**
9. If y varies directly as x and $y = 39$ when $x = 52$, find y when $x = 22$. **16.5**
10. If y varies directly as x and $x = 60$ when $y = 75$, find x when $y = 42$. **33.6**
11. Suppose y varies jointly as x and z. Find y when $x = 7$ and $z = 18$, if $y = 351$ when $x = 6$ and $z = 13$. **567**
12. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 27$, if $y = 480$ when $x = 9$ and $z = 20$. **360**

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NAME ______________________________ DATE ____________ PERIOD _____

8-4 Study Guide and Intervention *(continued)*

Direct, Joint, and Inverse Variation

Inverse Variation

Inverse Variation	y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

Example **If a varies inversely as b and $a = 8$ when $b = 12$, find a when $b = 4$.**

$\frac{a_1}{b_2} = \frac{a_2}{b_1}$ Inverse variation

$\frac{8}{4} = \frac{a_2}{12}$ $a_1 = 8$, $b_1 = 12$, $b_2 = 4$

$8(12) = 4a_2$ Cross multiply.

$96 = 4a_2$ Simplify.

$24 = a_2$ Divide each side by 4.

When $b = 4$, the value of a is 24.

Exercises

Find each value.

1. If y varies inversely as x and $y = 12$ when $x = 10$, find y when $x = 15$. **8**
2. If y varies inversely as x and $y = 100$ when $x = 38$, find y when $x = 76$. **50**
3. If y varies inversely as x and $y = 32$ when $x = 42$, find y when $x = 24$. **56**
4. If y varies inversely as x and $y = 36$ when $x = 10$, find y when $x = 30$. **12**
5. If y varies inversely as x and $y = 18$ when $x = 124$, find y when $x = 93$. **24**
6. If y varies inversely as x and $y = 90$ when $x = 35$, find y when $x = 50$. **63**
7. If y varies inversely as x and $y = 42$ when $x = 48$, find y when $x = 36$. **56**
8. If y varies inversely as x and $y = 44$ when $x = 20$, find y when $x = 55$. **16**
9. If y varies inversely as x and $y = 80$ when $x = 14$, find y when $x = 35$. **32**
10. If y varies inversely as x and $y = 3$ when $x = 8$, find y when $x = 40$. **0.6**
11. If y varies inversely as x and $y = 16$ when $x = 42$, find y when $x = 14$. **48**
12. If y varies inversely as x and $y = 23$ when $x = 12$, find y when $x = 15$. **18.4**

Lesson 8-4

Answers

NAME ______________________ DATE ____________ PERIOD _____

8-4 Skills Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $c = 12m$ **direct; 12**
2. $p = \frac{4}{q}$ **inverse; 4**
3. $A = \frac{1}{2}bh$ **joint; $\frac{1}{2}$**
4. $rw = 15$ **inverse; 15**
5. $y = 2rst$ **joint; 2**
6. $f = 5280m$ **direct; 5280**
7. $y = 0.2s$ **direct; 0.2**
8. $vz = -25$ **inverse; −25**
9. $t = 16rh$ **joint; 16**
10. $R = \frac{8}{w}$ **inverse; 8**
11. $\frac{a}{b} = \frac{1}{3}$ **direct; $\frac{1}{3}$**
12. $C = 2\pi r$ **direct; 2π**

Find each value.

13. If y varies directly as x and $y = 35$ when $x = 7$, find y when $x = 11$. **55**

14. If y varies directly as x and $y = 360$ when $x = 180$, find y when $x = 270$. **540**

15. If y varies directly as x and $y = 540$ when $x = 10$, find x when $y = 1080$. **20**

16. If y varies directly as x and $y = 12$ when $x = 72$, find x when $y = 9$. **54**

17. If y varies jointly as x and z and $y = 18$ when $x = 2$ and $z = 3$, find y when $x = 5$ and $z = 6$. **90**

18. If y varies jointly as x and z and $y = -16$ when $x = 4$ and $z = 2$, find y when $x = -1$ and $z = 7$. **14**

19. If y varies jointly as x and z and $y = 120$ when $x = 4$ and $z = 6$, find y when $x = 3$ and $z = 2$. **30**

20. If y varies inversely as x and $y = 2$ when $x = 2$, find y when $x = 1$. **4**

21. If y varies inversely as x and $y = 6$ when $x = 5$, find y when $x = 10$. **3**

22. If y varies inversely as x and $y = 3$ when $x = 14$, find x when $y = 6$. **7**

23. If y varies inversely as x and $y = 27$ when $x = 2$, find x when $y = 9$. **6**

24. If y varies directly as x and $y = -15$ when $x = 5$, find x when $y = -36$. **12**

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NAME ______________________ DATE ____________ PERIOD _____

8-4 Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$ **joint; 8**
2. $p = 4s$ **direct; 4**
3. $L = \frac{5}{k}$ **inverse; 5**
4. $xy = 4.5$ **inverse; 4.5**
5. $\frac{C}{d} = \pi$ **direct; π**
6. $2d = mn$ **joint; $\frac{1}{2}$**
7. $\frac{1.25}{g} = h$ **inverse; 1.25**
8. $y = \frac{3}{4x}$ **inverse; $\frac{3}{4}$**

Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$. **24**

10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$. **1.5**

11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$. **396**

12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$. **$\frac{56}{3}$**

13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$. **288**

14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$. **240**

15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$. **8**

16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$. **$\frac{64}{3}$**

17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$. **6**

18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$. **−21.6**

19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$. **3**

20. GASES The volume V of a gas varies inversely as its pressure P. If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury. **500 cm^3**

21. SPRINGS The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached? **12 in.**

22. GEOMETRY The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters? **100 m^2**

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8-4 Word Problem Practice

Direct, Joint, and Inverse Variation

1. **DIVING** The height that a diver leaps above a diving board varies directly with the amount that the tip of the diving board dips below its normal level. If a diver leaps 44 inches above the diving board when the diving board tip dips 12 inches, how high will the diver leap above the diving board if the tip dips 18 inches?

66 inches

2. **PARKING LOT DESIGN** As a general rule, the number of parking spaces in a parking lot for a movie theater complex varies directly with the number of theaters in the complex. A typical theater has 30 parking spaces for each theater. A businessman wants to build a new cinema complex on a lot that has enough space for 210 parking spaces. How many theaters should the businessman build in his complex?

7

3. **RENT** An apartment rents for m dollars per month. If n students share the rent equally, how much would each student have to pay? How does the cost per student vary with the number of students? If 2 students have to pay $700 each, how much money would each student have to pay if there were 5 students sharing the rent?

Each student pays $\frac{m}{n}$ dollars.

The cost per student varies inversely with the number of students, so each student would pay $280.

4. **PAINTING** The cost of painting a wall varies directly with the area of the wall. Write a formula for the cost of painting a rectangular wall with dimensions ℓ by w. With respect to ℓ and w, does the cost vary directly, jointly, or inversely?

$C = k\ell w$, where C is the cost and k is a constant. C varies jointly with ℓ and w.

HYDROGEN For Exercises 5-7, use the following information.

The cost of a hydrogen storage tank varies directly with the volume of the tank. A laboratory wants to purchase a storage tank shaped like a block with dimensions L by W by H.

5. Fill in the missing spaces in the following table from a brochure of various tank sizes.

Hydrogen Tank Dimensions (inches)			Cost
L	*W*	*H*	
36	36	36	**$900**
18	**18**	24	$150
24	24	72	$800

6. The hydrogen tank must fit in a shelf that has a fixed height and depth. How does the cost of the hydrogen storage tank vary with the width of tank with fixed depth and height?

The cost varies directly with the width.

7. How much would a spherical tank of radius 24 inches cost? (Recall that the volume of a sphere is given by $\frac{4}{3}\pi r^3$, where r is the radius.)

$1,117.01

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8-4 Enrichment

Geosynchronous Satellites

Satellites circling the Earth are almost as common as the cell phones that depend on them. A geosynchronous satellite is one that maintains the same position above the Earth at all times. Geosynchronous satellites are used in cell phone communications, transmitting signals from towers on Earth and to each other.

The speed at which they travel is very important. If the speed is too low, the satellite will be forced back down to Earth due to the Earth's gravity. However, if it is too fast, it will overcome gravity's force and escape into space, never to return. Newton's second law of motion says that force on an object is equal to mass times acceleration or $F = ma$. It is also well known that the net gravitational force between two objects is inversely proportional to the square of the distance between them. Therefore, there are two variables on which the force depends: speed and height above the Earth.

In particular, Newton's second law, $F = ma$, shows that force varies directly with acceleration, where m is the constant taking the place of "k."

Exercises

1. Show that the net gravitational force providing a satellite with acceleration is inversely proportional to the square of the distance between them by expressing this variation as an equation.

$F = \frac{k}{h^2}$, where h is the height of the satellite above the surface of the Earth.

2. Use your equation from Number 1 and equate it with Newton's formula above to determine how the satellite's acceleration varies with its height above the Earth.

$ma = \frac{k}{h^2} \Rightarrow a = \frac{k}{m} \cdot \frac{1}{h^2} = \frac{K}{h^2}$, therefore it varies inversely with the square of the height.

3. Determine how the speed of a geosynchronous satellite varies with its height above the Earth by using the fact that speed is equal to distance divided by time and the path of the satellite is circular.

Direct variation. $speed = \frac{distance}{time} \Rightarrow speed = \frac{2\pi r}{day}$, where $r = h +$ Radius of the Earth.

NAME ______________________ DATE __________ PERIOD _____

8-4 Spreadsheet Activity

Variation

You have learned to solve problems involving direct, inverse, and joint variation. Many physical situations involve at least one of these types of variation. For example, according to Newton's law of universal gravitation, the weight of a mass near Earth depends on the distance between the mass and the center of Earth. Study the spreadsheet below to determine the type of variation that exists between the quantity of an astronaut's weight and the distance of the astronaut from the center of Earth.

In the spreadsheet, the values for the astronaut's weight in newtons are entered in the cells in column A, and the values for the astronaut's distance in meters from the center of Earth are entered in cells in column B. Column C contains the astronaut's distance from Earth's surface.

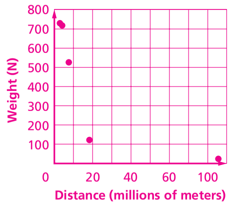
Gravitation.xls

◇	A	B	C
1	Astronaut's Weight (N)	Distance from Earth's Center (m)	Distance from Earth's Surface (km)
2	734.5843	6,380,000	0
3	712.0675	6,480,000	100
4	548.9825	7,380,000	1000
5	111.4406	16,380,000	10,000
6	2.642112	106,380,000	100,000
7			

Sheet 1 | Sheet 2 | Sheet 3

Exercises

1. Use the values in the spreadsheet to make a graph of the astronaut's weight plotted against the astronaut's distance from Earth's center.



2. Based on your graph, is this an inverse or direct variation? **inverse**

3. Write an equation that represents this situation. Let W represent the astronaut's weight, k the constant of variation, and R the distance from Earth's center. $W = \frac{K}{R^2}$

4. Use the equation to find the weight of the astronaut at these distances from Earth's surface. (*Hint*: Remember to add these values to the value in cell B2 to find the distance from Earth's center.)

a. 145,300,000 m **1.299615 N**
b. 65 m **734.5494 N**
c. 25,600 m **728.7047 N**
d. 300,800,700 m **0.316872 N**
e. 6580 m **733.0515 N**
f. 180,560 m **694.6873 N**

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8-5 Lesson Reading Guide

Classes of Functions

Get Ready for the Lesson

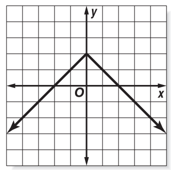
Read the introduction to Lesson 8-5 in your textbook.

- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.
  about 15 pounds
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.
  about 45 pounds

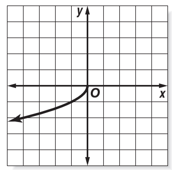
Read the Lesson

1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

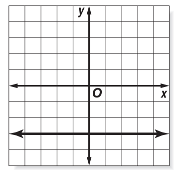
I. square root **II.** quadratic **III.** absolute value **IV.** rational
V. greatest integer **VI.** constant **VII.** identity

a. 

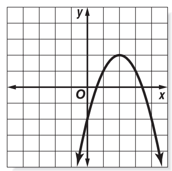
III

b. 

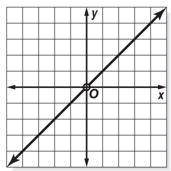
I

c. 

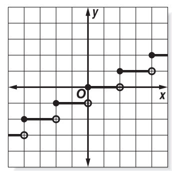
VI

d. 

II

e. 

IV

f. 

V

Remember What You Learned

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$? **Sample answer: Using the definition of absolute value, $f(x) = x$ if $x \geq 0$ and $f(x) = -x$ if $x < 0$. Therefore, the graph is made up of pieces of two lines, one with slope 1 and one with slope −1, meeting at the origin. This forms a V-shaped graph with "vertex" at the origin.**

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Lesson 8-5

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8-5 Study Guide and Intervention

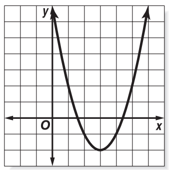
Classes of Functions

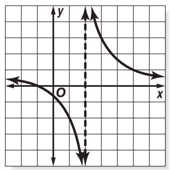
Identify Graphs You should be familiar with the graphs of the following functions.

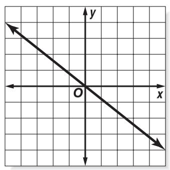
Function	Description of Graph
Constant	a horizontal line that crosses the *y*-axis at *a*
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (*a*, *a*), where *a* is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

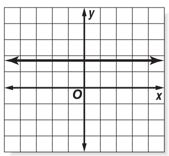
Exercises

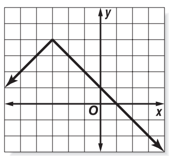
Identify the function represented by each graph.

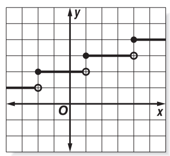
1.

quadratic

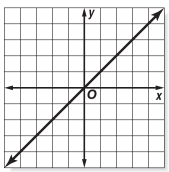
2.

rational

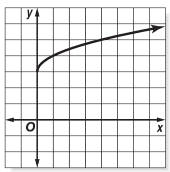
3.

direct variation

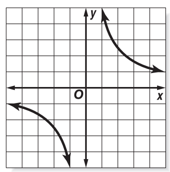
4.

constant

5.

absolute value

6.

greatest integer

7.

identity

8.

square root

9.

inverse variation

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8-5 Study Guide and Intervention *(continued)*

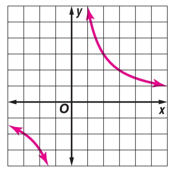
Classes of Functions

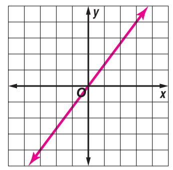
Identify Equations You should be able to graph the equations of the following functions.

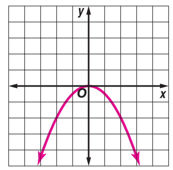
Function	General Equation
Constant	$y = a$
Direct Variation	$y = ax$
Greatest Integer	equation includes a variable within the greatest integer symbol, [[ ]]
Absolute Value	equation includes a variable within the absolute value symbol, | |
Quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
Square Root	equation includes a variable beneath the radical sign, $\sqrt{\ }$
Rational	$y = \frac{p(x)}{q(x)}$
Inverse Variation	$y = \frac{a}{x}$

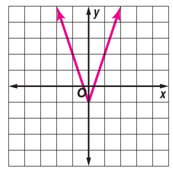
Exercises

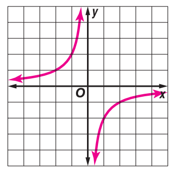
Identify the function represented by each equation. Then graph the equation.

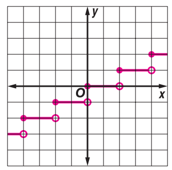
1. $y = \frac{6}{x}$ inverse variation


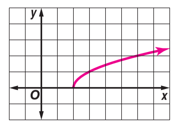
2. $y = \frac{4}{3}x$ direct variation


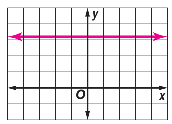
3. $y = -\frac{x^2}{2}$ quadratic


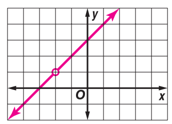
4. $y = |3x| - 1$ absolute value


5. $y = -\frac{2}{x}$ inverse variation


6. $y = \left[\!\left[\frac{x}{2}\right]\!\right]$ greatest integer


7. $y = \sqrt{x - 2}$ square root


8. $y = 3.2$ constant


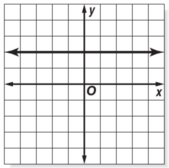
9. $y = \frac{x^2 + 5x + 6}{x + 2}$ rational


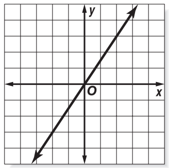
NAME ______________________ DATE __________ PERIOD _____

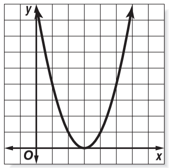
8-5 Skills Practice

Classes of Functions

Identify the type of function represented by each graph.

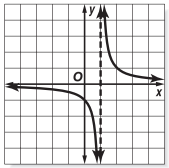
1.

constant

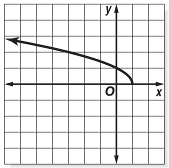
2.

direct variation

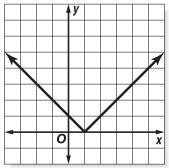
3.

quadratic

Match each graph with an equation below.

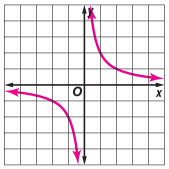
A. $y = |x - 1|$ **B.** $y = \frac{1}{x - 1}$ **C.** $y = \sqrt{1 - x}$ **D.** $y = [\![x]\!] - 1$

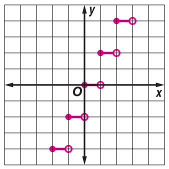
4. B


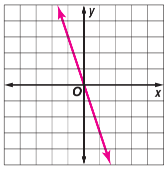
5. C


6. A


Identify the type of function represented by each equation. Then graph the equation.

7. $y = \frac{2}{x}$

inverse variation or rational

8. $y = 2[\![x]\!]$

greatest integer

9. $y = -3x$

direct variation

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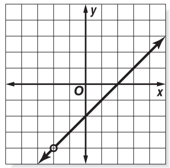
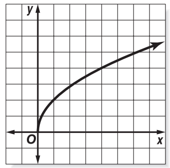
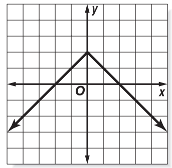
8-5 Practice

Classes of Functions

Identify the type of function represented by each graph.

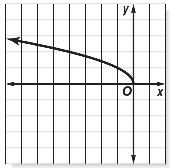
1.

rational

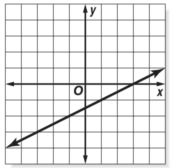
2.

square root

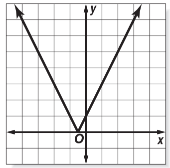
3.

absolute value

Match each graph with an equation below.

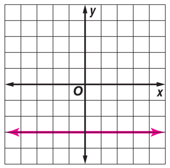
A. $y = |2x + 1|$ **B.** $y = [\![2x + 1]\!]$ **C.** $y = \frac{x - 3}{2}$ **D.** $y = \sqrt{-x}$

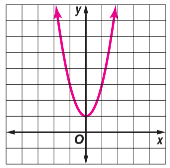
4. D


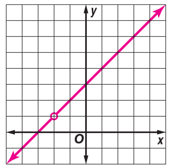
5. C


6. A


Identify the type of function represented by each equation. Then graph the equation.

7. $y = -3$

constant

8. $y = 2x^2 + 1$

quadratic

9. $y = \frac{x^2 + 5x + 6}{x + 2}$

rational

10. BUSINESS A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function. The graph is U-shaped; it is a parabola.

11. PARKING A parking lot charges $10 to park for the first day or part of a day. After that, it charges an additional $8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days. The graph looks like a series of steps, similar to a greatest integer function, but with open circles on the left and closed circles on the right; $58.

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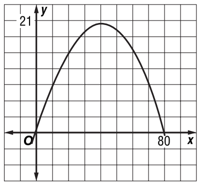
8-5 Word Problem Practice

Classes of Functions

1. **STAIRS** What type of a function has a graph that could be used to model a staircase?

   the greatest integer function

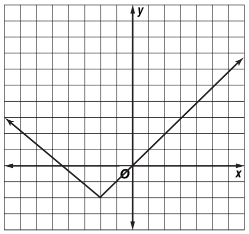
2. **GOLF BALLS** The trajectory of a golf ball hit by an astronaut on the moon is described by the function $f(x) = -0.0125(x - 40)^2 + 20$.



   Describe the shape of this trajectory.

   a parabola

3. **RAVINE** The graph shows the cross-section of a ravine.



   What kind of function is represented by the graph? Write the function.

   an absolute value function; $f(x) = |x + 2| - 2$

4. **LEAKY FAUCETS** A leaky faucet leaks 1 milliliter of water every second. Write a function that gives the number of milliliters leaked in t seconds as a function of t. What type of function is it?

   $f(t) = t$; an identity function

PUBLISHING For Exercises 5-8, use the following information.

Kate has just finished writing a book that explains how to sew your own stuffed animals. She hopes to make $1000 from sales of the book because that is how much it would cost her to go to the European Sewing Convention. Each book costs $2 to print and assemble. Let P be the selling price of the book. Let N be the number of people who will buy the book.

5. Write an equation that relates P and N if she earns exactly $1,000 from sales of the book.

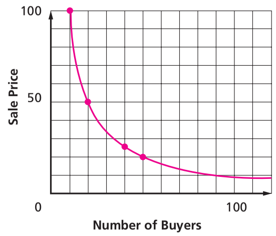
   $1000 = N(P - 2)$

6. Solve the equation you wrote for Exercise 5 for P in terms of N.

   $P = \dfrac{2N + 1000}{N}$

7. What kind of function is P in terms of N? Sketch a graph of P as a function of N.

   rational;



8. If Kate thinks that 125 people will buy her book, how much should she charge for the book?

   $10

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8-5 Enrichment

Physical Properties of Functions

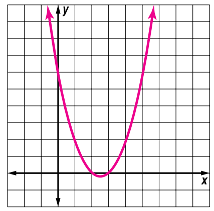
Mathematical functions are classified based on properties similar to how biologists classify animal species. Functions can be classified as continuous or non-continuous, increasing or decreasing, polynomial or non-polynomial for example. The class of polynomials functions can be further classified as linear, quadratic, cubic, etc., based on its *degree*.

Characteristics of functions include:
- A function is **bounded below** if there exists a number that is less than any function value.
- A function is **bounded above** if a number exists that is greater than any function value.
- A function is **symmetric** (about a vertical axis) if it is a mirror image about that vertical axis.
- A function is **continuous** if it can be drawn without lifting your pencil.
- A function is **increasing** if $f(x) > f(y)$ when $x > y$. Continual growth from left to right.
- A function is **decreasing** if $f(x) < f(y)$ when $x < y$. Continual decay from left to right.

Exercises

1. Sketch the graph of $y = x^2 - 5x + 6$. List the characteristics of functions displayed by this graph.

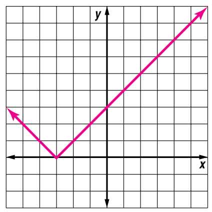
   Some properties include: symmetric, continuous, bounded below.



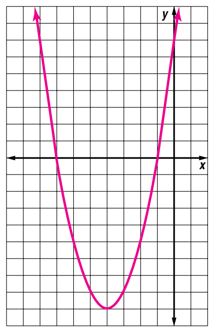
2. What characteristics do absolute value functions and quadratic functions have in common? How do they differ?

   Common: Symmetric, continuous, bounded below (or above). Differ: One is U shaped and the other V shaped, one is smooth and one has a rigid corner, and one increases more rapidly than the other.

3. Graph $y = |x + 3|$.



4. Graph $y = x^2 + 8x + 7$.



NAME ______________ DATE ________ PERIOD ____

8-6 Lesson Reading Guide

Solving Rational Equations and Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 8-6 in your textbook.

- If you increase the number of songs that you download, will your total bill increase or decrease? **increase**
- Will your actual cost per song increase or decrease? **decrease**

Read the Lesson

1. When solving a rational equation, any possible solution that results in **0** in the denominator must be excluded from the list of solutions.

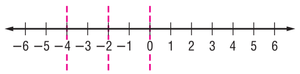
2. Suppose that on a quiz you are asked to solve the rational inequality $\frac{3}{z+2} - \frac{6}{z} > 0$. Complete the steps of the solution.

Step 1 The excluded values are **−2** and **0**.

Step 2 The related equation is $\mathbf{\frac{3}{z+2} - \frac{6}{z} = 0}$.

To solve this equation, multiply both sides by the LCD, which is $\mathbf{z(z+2)}$. Solving this equation will show that the only solution is −4.

Step 3 Divide a number line into **4** regions using the excluded values and the solution of the related equation. Draw dashed vertical lines on the number line below to show these regions.



Consider the following values of $\frac{3}{z+2} - \frac{6}{z}$ for various test values of z.

If $z = -5$, $\frac{3}{z+2} - \frac{6}{z} = 0.2$. If $z = -3$, $\frac{3}{z+2} - \frac{6}{z} = -1$.

If $z = -1$, $\frac{3}{z+2} - \frac{6}{z} = 9$. If $z = 1$, $\frac{3}{z+2} - \frac{6}{z} = -5$.

Using this information and your number line, write the solution of the inequality.

$\mathbf{z < -4 \text{ or } -2 < z < 0}$

Remember What You Learned

3. How are the processes of adding rational expressions with different denominators and of solving rational expressions alike, and how are they different? **Sample answer: They are alike because both use the LCD of all the rational expressions in the problem. They are different because in an addition problem, the LCD remains after the fractions are added, while in solving a rational equation, the LCD is eliminated.**

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8-6 Study Guide and Intervention

Solving Rational Equations and Inequalities

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$	Multiply each side by 10(x + 1).
$9(x+1) + 2(10) = 4(x+1)$	Multiply.
$9x + 9 + 20 = 4x + 4$	Distributive Property
$5x = -25$	Subtract 4x and 29 from each side.
$x = -5$	Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$	$x = -5$
$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{2}{5} = \frac{2}{5}$	

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$ **5**
2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$ **2**
3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$ $\mathbf{-\frac{13}{5}}$
4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$ $\mathbf{-\frac{1}{24}}$
5. $\frac{4}{x-1} = \frac{x+1}{12}$ **±7**
6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$ $\mathbf{\frac{8}{3}}$

7. **NAVIGATION** The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water? Is this a reasonable answer? Explain. **42 mph; Sample answer: The answer is reasonable. The boat will travel 48 mph one way and 36 mph the other way. Therefore it will take $\frac{1}{3}$ of an hour to travel 16 miles and 12 miles, respectively.**

8. **WORK** Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together? Is this a reasonable answer? **about $1\frac{2}{3}$ days; Sample answer: It is a reasonable answer. It will take each person about 5 days to paint the house alone, so it should take about $\frac{1}{3}$ of the time to paint the house together.**

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8-6 Study Guide and Intervention *(continued)*

Solving Rational Equations and Inequalities

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

Step 1 State the excluded values.
Step 2 Solve the related equation.
Step 3 Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

Example **Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.**

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$ Related equation

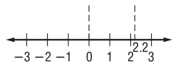
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$ Multiply each side by $15n$.

$10 + 12 = 10n$ Simplify.

$22 = 10n$ Simplify.

$2.2 = n$ Simplify.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$.
$-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3}$ is true.

Test $n = 1$.
$\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3}$ is *not* true.

Test $n = 3$.
$\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \geq 2.2$.

Exercises

Solve each inequality.

1. $\frac{3}{a+1} \geq 3$ **$-1 < a \leq 0$**

2. $\frac{1}{x} \geq 4x$ **$x \leq -\frac{1}{2}$ or $0 < x \leq \frac{1}{2}$**

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$ **$0 < p < \frac{39}{20}$**

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$ **$-2 < x < 0$**

5. $\frac{4}{x-1} + \frac{5}{x} < 2$ **$x < 0$ or $\frac{1}{2} < x < 1$ or $x > 5$**

6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$ **$x < -1$ or $0 < x < 1$ or $x > 2$**

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8-6 Skills Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$ **-1**

2. $2 = \frac{4}{n} + \frac{1}{3}$ **$\frac{12}{5}$**

3. $\frac{9}{3x} = \frac{-6}{2}$ **-1**

4. $3 - z = \frac{2}{z}$ **1, 2**

5. $\frac{2}{d+1} = \frac{1}{d-2}$ **5**

6. $\frac{s-3}{5} = \frac{8}{s}$ **$-5, 8$**

7. $\frac{2x+3}{x+1} = \frac{3}{2}$ **-3**

8. $-\frac{12}{y} = y - 7$ **3, 4**

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$ **8**

10. $\frac{3}{k} - \frac{4}{3k} > 0$ **$k > 0$**

11. $2 - \frac{3}{v} < \frac{5}{v}$ **$0 < v < 4$**

12. $n + \frac{3}{n} < \frac{12}{n}$ **$n < -3$ or $0 < n < 3$**

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$ **$0 < m < 1$**

14. $\frac{1}{2x} < \frac{2}{x} - 1$ **$0 < x < \frac{3}{2}$**

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$ **3**

16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$ **4**

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$ **-5**

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$ **$\frac{2}{5}$**

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$ **-4**

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$ **$\varnothing$**

21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$ **$\varnothing$**

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$ **2**

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$ **-6**

24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$ **5**

NAME ______________ DATE ________ PERIOD _____

8-6 Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ **16**

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ **−1, 2**

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ **$-\frac{2}{3}$, 4**

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ **4**

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ **all reals except 5**

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ **$\frac{5}{8}$**

7. $\frac{5}{t} < \frac{9}{2t+1}$ **$t < -5$ or $-\frac{1}{2} < t < 0$**

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ **$\frac{11}{5}$**

9. $\frac{4}{w-2} = \frac{-1}{w+3}$ **−2**

10. $5 - \frac{3}{a} < \frac{7}{a}$ **$0 < a < 2$**

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ **$0 < x < 7$**

12. $8 + \frac{3}{y} > \frac{19}{y}$ **$y < 0$ or $y > 2$**

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ **$p < 0$ or $p > \frac{65}{3}$**

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ **$\varnothing$**

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ **−1**

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ **−2**

17. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ **$\frac{3}{2}$**

18. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ **$-\frac{5}{3}$, 5**

19. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ **7**

20. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ **−1, −2**

21. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ **0**

22. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ **$\varnothing$**

23. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ **all reals except −4 and 4**

24. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ **−2**

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? Is this a reasonable answer? Explain. **6; Sample answer: It is a reasonable answer. She will have made 15 out of 25 free throws, which is equivalent to 60%.**

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? Is this a reasonable answer? Explain. **20 cm; Sample answer: It is a reasonable answer, since $\frac{1}{20} + \frac{1}{5} = \frac{1}{4}$.**

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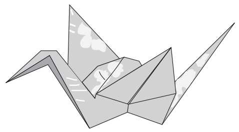
8-6 Word Problem Practice

Solving Rational Equations and Inequalities

1. **HEIGHT** Serena can be described as being 8 inches shorter than her sister Malia, or as being 12.5% shorter than Malia. In other words, $\frac{8}{H+8} = \frac{1}{8}$, where H is Serena's height in inches. How tall is Serena?
56 inches

2. **CRANES** For a wedding, Paula wants to fold 1000 origami cranes.



She does not want to make anyone fold more than 15 cranes. In other words, if N is the number of people enlisted to fold cranes, Paula wants $\frac{1000}{N} \le 15$. What is the minimum number of people that will satisfy this inequality?
67

3. **RENTAL** Carlos and his friends rent a car. They split the $200 rental fee evenly. Carlos, together with just two of his friends, decide to rent a portable DVD player as well, and split the $30 rental fee for the DVD player evenly among themselves. Carlos ends up spending $50 for these rentals. Write an equation involving N, the number of friends Carlos has, using this information. Solve the equation for N.
$\frac{200}{N+1} + \frac{30}{3} = 50$; $N = 4$

4. **PROJECTILES** A projectile target is launched into the air. A rocket interceptor is fired at the target. The ratio of the altitude of the rocket to the altitude of the projectile t seconds after the launch of the rocket is given by the formula $\frac{333t}{-32t^2 + 420t + 27}$. At what time are the target and interceptor at the same altitude?
at $t = 3$ seconds

FLIGHT TIME For Exercises 5 and 6, use the following information.

The distance between New York City and Los Angeles is about 2500 miles. Let S be the airspeed of a jet. The wind speed is 100 miles per hour. Because of the wind, it takes longer to fly one way than the other.

5. Write an equation for S if it takes 2 hours and 5 minutes longer to fly between New York and Los Angeles against the wind versus flying with the wind.
$\frac{2500}{S-100} - \frac{2500}{S+100} = 2\frac{1}{12}$

6. Solve the equation you wrote in Exercise 5 for S.
500 mph

7. Write an equation and find how much longer to fly between New York and Los Angeles if the wind speed increases to 150 miles per hour and the airspeed of the jet is 525 miles per hour.
$\frac{2500}{525-150} - \frac{2500}{525+150} = x$;
$x = \frac{400}{135} = 2.96$ h

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8-6 Enrichment

Oblique Asymptotes

The graph of $y = ax + b$, where $a \neq 0$, is called an oblique asymptote of $y = f(x)$ if the graph of f comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$. ∞ is the mathematical symbol for **infinity**, which means *endless*.

For $f(x) = 3x + 4 + \frac{2}{x}$, $y = 3x + 4$ is an oblique asymptote because $f(x) - 3x - 4 = \frac{2}{x}$, and $\frac{2}{x} \to 0$ as $x \to \infty$ or $-\infty$. In other words, as $|x|$ increases, the value of $\frac{2}{x}$ gets smaller and smaller approaching 0.

Example **Find the oblique asymptote for $f(x) = \frac{x^2 + 8x + 15}{x + 2}$.**

$$\begin{array}{r|rrr} -2 & 1 & 8 & 15 \\ & & -2 & -12 \\ \hline & 1 & 6 & 3 \end{array}$$

Use synthetic division.

$$y = \frac{x^2 + 8x + 15}{x + 2} = x + 6 + \frac{3}{x + 2}$$

As $|x|$ increases, the value of $\frac{3}{x + 2}$ gets smaller. In other words, since $\frac{3}{x + 2} \to 0$ as $x \to \infty$ or $x \to -\infty$, $y = x + 6$ is an oblique asymptote.

Use synthetic division to find the oblique asymptote for each function.

1. $y = \frac{8x^2 - 4x + 11}{x + 5}$ **$y = 8x - 44$**

2. $y = \frac{x^2 + 3x - 15}{x - 2}$ **$y = x + 5$**

3. $y = \frac{x^2 - 2x - 18}{x - 3}$ **$y = x + 1$**

4. $y = \frac{ax^2 + bx + c}{x - d}$ **$y = ax + b + ad$**

5. $y = \frac{ax^2 + bx + c}{x + d}$ **$y = ax + b - ad$**

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